TRINITY COLLEGE

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

The level of Strontium-90 in a contaminated soil sample at the start of 1985 was 0.65 mg/kg. Strontium-90 has a half-life of 28.2 years and decays continuously such that $S = S_0 e^{kt}$ where S is the level of Strontium-90, t is the time in years since the level was S_0 and k is a constant.

(a) Assuming no further contamination occurred, determine

(i) the level of Strontium-90 in the sample at the start of 2018. (3 marks)

(ii) the rate of change of the level of Strontium-90 in the sample at the start of 2018. (1 mark)

(b) Strontium-90 decays into Yttrium-90. The mass of Yttrium-90 decays continuously such that $Y = Y_0 e^{-0.0112t}$ where Y is the mass of Yttrium-90 and t is the time in hours since the level was Y_0. Determine the time taken for a mass of Yttrium-90 to decrease by 80%. (2 marks)

Question 10 **(8 marks)**

The discrete random variable X has $\mathrm{E}(X) = 3.2$ and probability function

$$P(X = x) = \begin{cases} a + bx & x = 2, 3, 4 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine the values of the constants a and b. (4 marks)

(b) Determine $\mathrm{Var}(X)$. (2 marks)

(c) A second random variable Y is a linear transformation of X such that $Y = kX + 4$, where k is a constant and $\mathrm{E}(Y) = 20$. Determine $\mathrm{Var}(Y)$. (2 marks)

See next page

Question 11 **(6 marks)**

A retail chain wants to know what proportion of its customers support a recent decision to extend the range of clothes sold at its 18 stores.

(a) Comment, with reasons, on whether the following sampling methods are likely to introduce bias.

(i) Send an employee to one randomly selected store at noon on a Friday and get them to record the responses of the first 15 customers who arrive. (2 marks)

(ii) In a newsletter sent to all customers, include a link to a public page on their website where users can click a 'yes' or 'no' button to register their support. (2 marks)

(b) Following the analysis of a large random sample of customers, the business reported that the 99% confidence interval for customer support was from 0.8 to 0.9. Mark each of the statements below as **true** or **false**, where false means that the statement does not follow logically from the report.

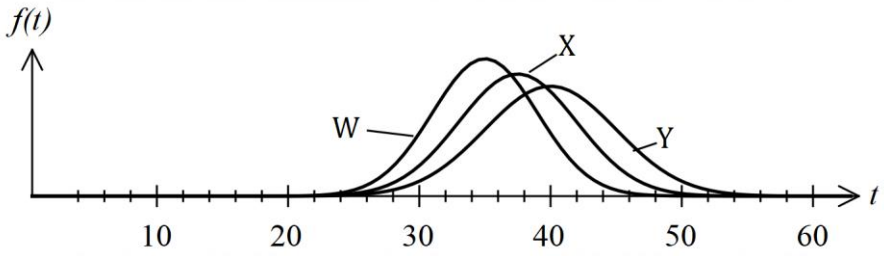
(i) If the random sampling was repeated over and over, then 99% of the time the true proportion of supportive customers will fall between 0.8 and 0.9. (1 mark)

(ii) There is a 99% chance that the true proportion of supportive customers lies between 0.8 and 0.9. (1 mark)

See next page

Question 12 **(8 marks)**

(a) The graphs of the probability density functions of three normally distributed random variables W, X and Y are shown below.



State, with justification, which of the three random variables has

(i) the largest standard deviation? (1 mark)

(ii) the largest mean? (1 mark)

(b) Empty bottles are filled with A mL of water, where A is a normally distributed random variable with mean of 510 mL and standard deviation of 7.5 mL.

(i) Determine the probability that a bottle is filled with more than 520 mL. (1 mark)

(ii) Determine the probability that a bottle is filled with less than 515 mL, given that it is filled with more than 510 mL. (2 marks)

(iii) The mean of A is to be decreased by k mL so that just 2.5% of bottles are filled with 520 mL or more. Determine the value of k. (3 marks)

See next page

Question 13 **(8 marks)**

235 out of a random sample of 855 people in a city had seen their dentist in the last year.

(a) If there were 219 000 people living in the city, estimate the actual number of these who had seen their dentist in the last year. (2 marks)

(b) Determine the approximate margin of error for a 99% confidence interval for the proportion of people who had seen their dentist in the last year. (2 marks)

(c) Determine an approximate 99% confidence interval for the true proportion of people who had seen their dentist in the last year. (2 marks)

(d) In order to confirm the sample proportion obtained from the random sample, another sample is to be taken. Estimate, to the nearest 10 people, the sample size required to obtain an approximate margin of error for a 99% confidence interval that is close to 0.07. (2 marks)

See next page

Question 14 **(10 marks)**

Every day a fisheries researcher randomly catches 10 fish from an inland lake containing a large number of fish, 68% of which are thought to be perch.

(a) The random variable X is the number of perch in the daily catch.

(i) Describe the distribution of X. (2 marks)

(ii) Over a period of 15 days, how many times would you expect the daily catch to contain more perch than fish of other species? (2 marks)

(iii) Determine the probability that a total of 19 perch are caught over two consecutive days. (2 marks)

(b) The researcher suspected that the proportion of perch was lower than initially thought, but more than 60%.

(i) Calculate an approximate 90% confidence interval for the proportion of perch in the lake given that over a 7-day period, a total of 49 perch were caught. (2 marks)

(ii) Use the confidence interval to comment on the researcher's suspicion. (2 marks)

See next page

Question 15 **(7 marks)**

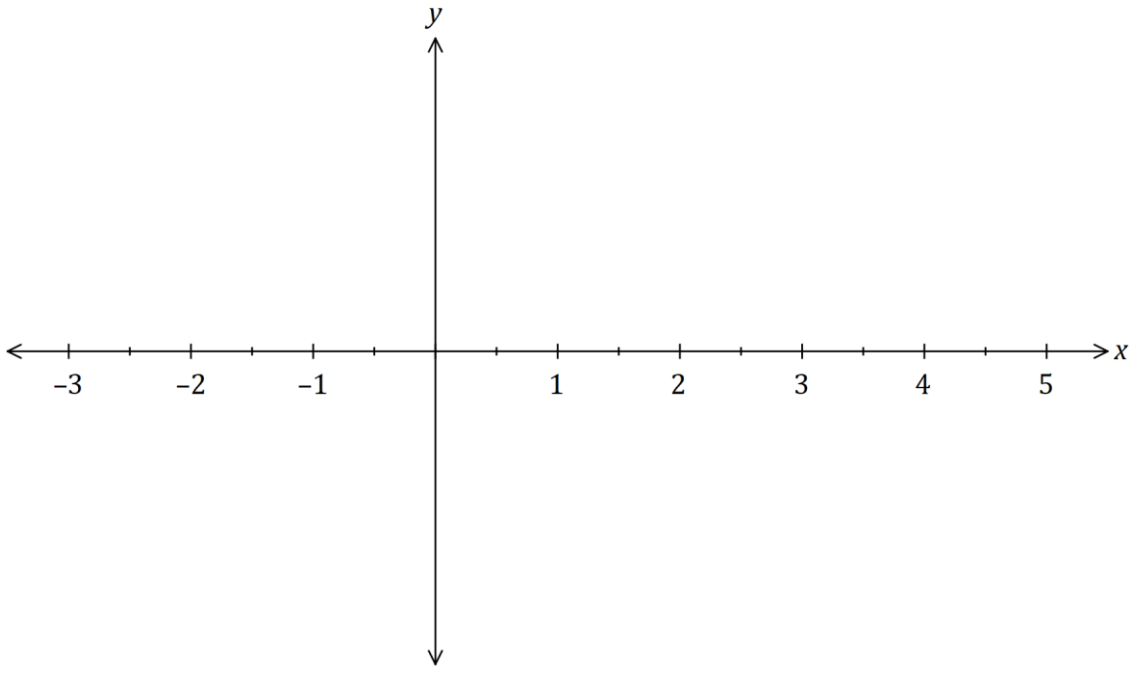
The table below shows the sign of the polynomial $f(x)$ and some of its derivatives at various values of x. There are no other zeroes of $f(x), f'(x)$ or $f''(x)$ apart from those shown in the table.

x	-2	-1	0	1	2	3	4
$f(x)$	$+$	0	$-$	$-$	$-$	0	$+$
$f'(x)$	$-$	$-$	0	$+$	$+$	0	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$	0	$+$

(a) For what value(s) of x is the graph of the function concave up? (1 mark)

(b) At what location does the graph of f have a turning point? Explain your answer. (2 marks)

(c) Sketch a possible graph of $y = f(x)$ on the axes below. (4 marks)



See next page

Question 16 **(8 marks)**

A student repeatedly took random samples of size 150 from a large population in which it was known that 38% of people were classified as overweight. For each sample, the proportion of overweight people was calculated and recorded as the sample proportion.

(a) Use an appropriate binomial distribution to determine the probability that the sample proportion is no more than 0.34 in a randomly chosen sample. (3 marks)

(b) After recording a large number of sample proportions, the student used them to create a histogram from which the approximate normality of their distribution was evident.

(i) Determine the expected mean and standard deviation of the observed normal distribution. (2 marks)

(ii) Use this normal distribution to determine the probability that the sample proportion is no more than 0.34 in a randomly chosen sample. (1 mark)

(iii) Describe how the parameters calculated in (i) would change if the student took smaller random samples. (2 marks)

See next page

Question 17 **(7 marks)**

At time $t = 0$, a small body P is at the origin O and is moving with a velocity of 18 ms^{-1}. The acceleration of P for $t \geq 0$ is given by

$$a = \frac{-3}{\sqrt{t+4}} \text{ ms}^{-2}.$$

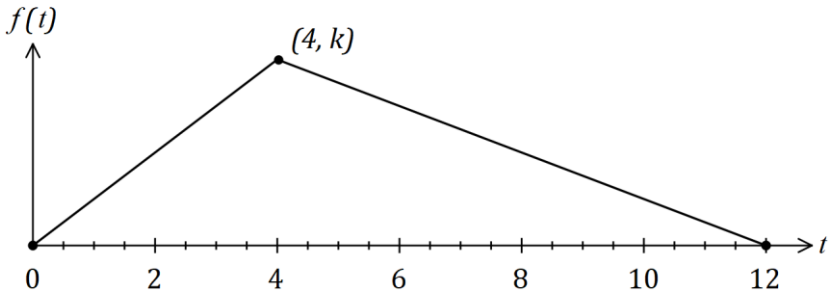
(a) Determine the velocity of P when $t = 5$. (4 marks)

(b) Determine the distance of P from O at the instant P is stationary. (3 marks)

See next page

Question 18 **(11 marks)**

The time T to process orders at a warehouse is a random variable which can take any value in the interval 0 to 12 minutes. The graph of the triangular probability density function of T is shown below.



(a) Determine the value of k. (1 mark)

(b) Determine the probability that the time to process an order takes less than 3 minutes. (3 marks)

See next page

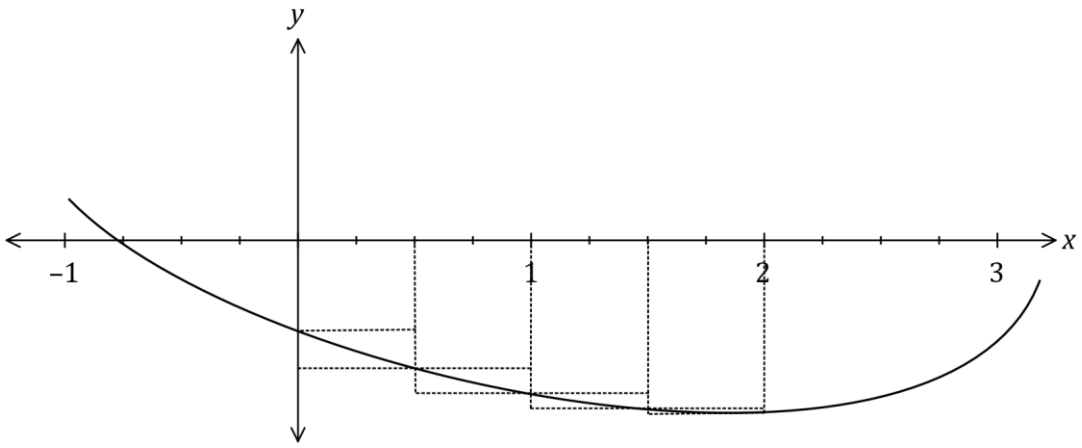
(c) Determine the mean time to process an order in minutes and seconds. (4 marks)

The variance of T is 6 minutes 13 seconds.

(d) Two new procedures will affect the processing time of an order. The first will decrease the time by 15% and the second will then add one-and-a-half minutes. Determine the new mean and standard deviation of the time to process an order. (3 marks)

Question 19 **(9 marks)**

(a) The graph of $y = f(x)$ is shown together with some values of $f(x)$.



x	-0.5	0	0.5	1	1.5	2	2.5
$f(x)$	-3.2	-6.6	-8.6	-11.1	-11.9	-12.2	-11.5

By considering the areas of the rectangles shown and using values of $f(x)$ from the table,

(i) calculate an underestimate for the numerical approximation for $\int_0^2 f(x)\,dx$.

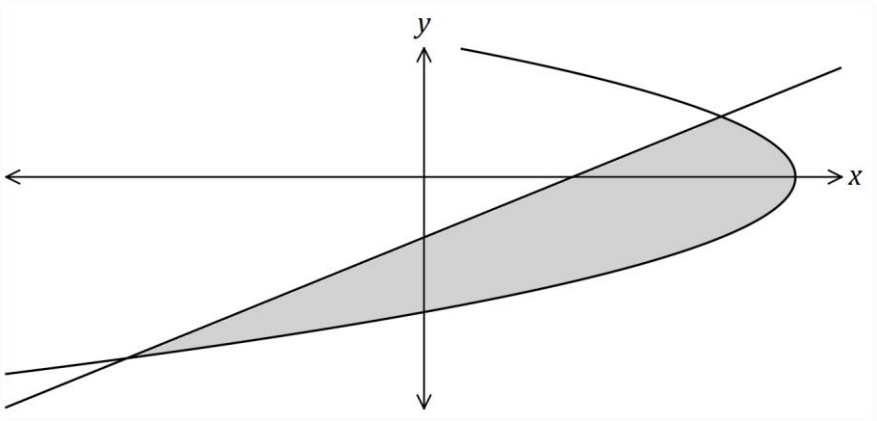
(2 marks)

(ii) calculate, using rectangles, a more accurate numerical approximation for $\int_0^2 f(x)\,dx$.

(3 marks)

See next page

(b) The graph of $x = 10 - 2y^2$ and the line $4y = x - 4$ are shown below.



Determine the area bounded by the line and the curve. (4 marks)

Question 20 **(6 marks)**

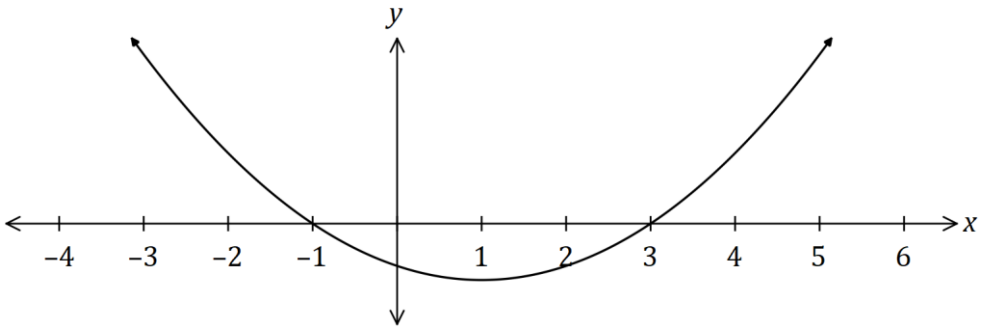
A game is played at a carnival where two fair 4-sided dice with faces numbered $1, 2, 3$ and 4 are tossed at the same time. Patrons pay $\$3$ for each play of the game, winning a major prize if both dice show a four or a minor prize if just one of the dice shows a four. The operator of the game buys major prizes for $\$22$ each, minor prizes for $\$2.50$ and must pay overhead costs of $\$95$ per day.

Determine how many times the game must be played per day so that the operator can expect to make a daily profit of at least $\$150$.

See next page

Question 21 **(4 marks)**

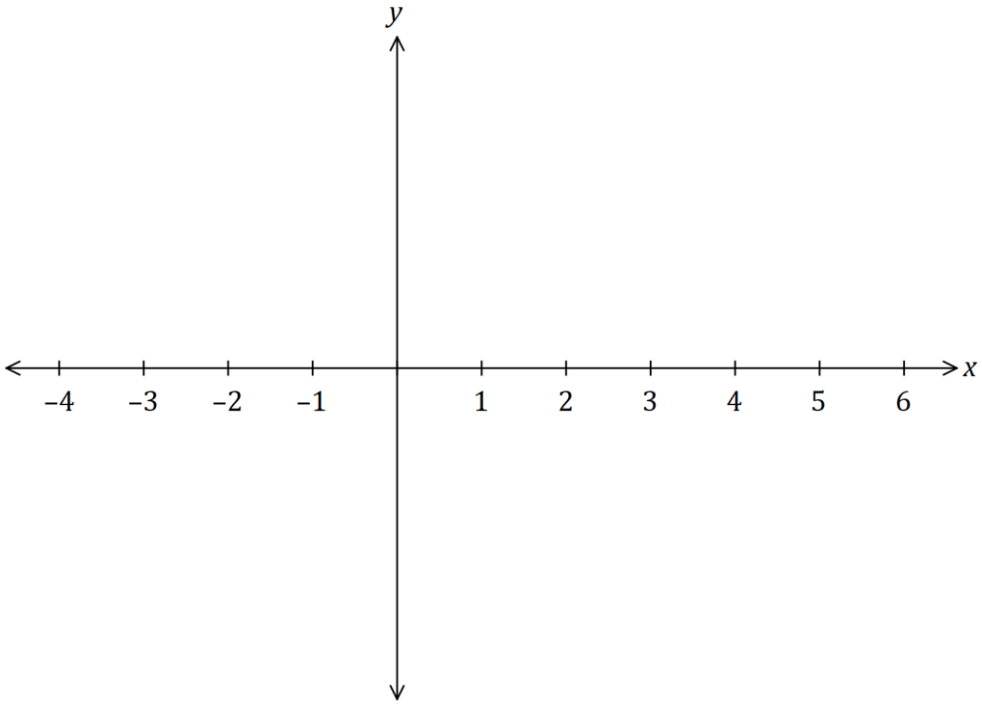
The graph of $y = f(x)$ is shown below.



Another function A is defined on the interval $-3 \leq x \leq 5$ by

$$A(x) = \int_{-3}^{x} f(t)\, dt.$$

It is known that $A(-1) = A(5) = 6$ and $A(3) = 0$. Sketch the graph of $y = A(x)$ on the axes below, clearly indicating the location of all x-intercepts, turning points, points of inflection and other key features.



End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____